Problem Solving and At-Risk Students:

Making "Mathematics for All" a Classroom Reality

Margot Fulton Robert

For two years, I taught in an affluent suburban school in one state where the children had the advantage of family and school experiences that encouraged their questions, ideas, and experimentation. Their experiences with books, travel, museums, dinner conversations, and school projects gave them confidence in themselves as thinkers. The school itself was committed to authentic project-based learning. We encouraged students to ask questions and think critically and to trust themselves and their classmates as inventors and problem solvers. After teaching a third- and fourth-grade class in this environment, I moved to a rural community in another state where 78 percent of the students live below the federal poverty level and where families struggle to meet the basic needs of their children. I came to this new job confident in my ability to structure learning environments, develop child-centered curricula, and meet the needs of diverse learners. I was in for a rude awakening. This group of students, now fifth graders, came to school with more anger and hunger than they did skills or enthusiasm. On more than one disheartening day, I found myself calling my brother in desperation: "I don't understand. I know I'm a good teacher, but nothing I try is working with these kids."

Margot Robert teaches fifth grade in Ossipee, NH 03814. She is interested in teaching students to be independent problem solvers in mathematics and in all aspects of their lives.

In that first year, everything I knew about teaching children to be problem solvers was brought into question. The methods that I had used successfully with high-achieving suburban students failed miserably in this new environment. When first confronted with this failure, I considered compromising the teaching of problem solving in favor of more basic mathematics skills learned by rote. Over the course of that year, I learned that with careful adaptations to my methods, these traditionally low achievers could become confident, and increasingly more competent, thinkers and problem solvers. Above all, I learned the importance of perseverance in challenging students, especially those whom many would consider to be at risk, until they experience for themselves the power of solving problems.

Early Discouragements

I began the school year with a design activity that all students love (or so I had thought). I gave each group of four students the same number of toothpicks, marshmallows, pieces of macaroni and spaghetti, and sticky labels. With this collection, the students were to work in teams to build a structure that was at least eight inches tall and would be judged by how much weight it could support (see **fig. 1**). A year earlier, my fourth graders had eagerly designed and built two-foot-high towers that held the weight of fifty nails.

The project had two objectives. First, I wanted the students to see that in fifth grade, we worked together, had fun, and gained the power to create things to be proud of. I also hoped that the project would reveal what the students knew about structures, how well they could make a plan and modify it as they progressed, and how well they could use one another's abilities. Both goals failed. To my surprise, not all students loved designing and building such a structure. Indeed, this group treated the activity with disdain. The students told me that the task was too hard and asked whether I would just tell them "the answer." Their tolerance for small failures was nonexistent. When a section of one group's tower fell the first time, instead of modifying the plan, the students declared the assignment "stupid." When we tested the towers' strength, some students seemed to want others' towers to fall instead of celebrating the group's success. I pulled the class together at the end of the activity to have groups share their designs and discuss the strengths of different structures. The dialogue that I intended to lead, which had been successful in the past, quickly disintegrated from a discussion of geometric shapes, angles, building materials, and architecture into wisecracks about smashed marshmallows. I went home profoundly discouraged. How would these students appreciate applied mathematics or scientific inquiry? Somewhere inside them, they must have the desire to create something "cool." Worrying about the problem was the first step in understanding the reaction of these fifth graders.

During that same week, I gave the group a classic problem:

> Farmer Joe went out one morning to check on his chickens and cows. Altogether, he saw 22 animals and 64 legs. How many of each animal (chickens and cows) did he see?

Figures 2 and **3** illustrate how students in my previous setting, with rich mathematical backgrounds from home and school, could rely on strategies to attack this problem. In giving this problem to my new fifth graders, my goal was to make a clearer assessment of their mathematical understanding and skills. Once again, I made the mistake of thinking that this kind of logic problem was fun. Seth tore his paper, Keith scribbled all over his, and Olivia and many others whined loudly, "I don't get it." David and Sumner put random answers on their papers, then provoked others; Harold serenaded us with the sounds of farm animals. The students' approach to the mathematics of this task illustrated the vast discrepancy between the skills that it required and the skills they had acquired. To determine the number of chickens and cows, students would need to use such strategies as drawing a picture and making a chart. The children also needed to start by interpreting the problem. This group of fifth graders lacked experience and fluency with any of these strategies. A few students tried to solve the problem by choosing an operation. In seeking help from me, they asked whether they should add 22 + 64 or multiply 22 × 64. In my eyes, they were not even trying to read and interpret the problem. From their perspective, mathematics had never made sense; why would this experience be any different?

Both the pasta tower and Farmer Joe's problem had threatened the students. When they declared

FIGURE 1

All hands must cooperate to design and build the strongest structure.

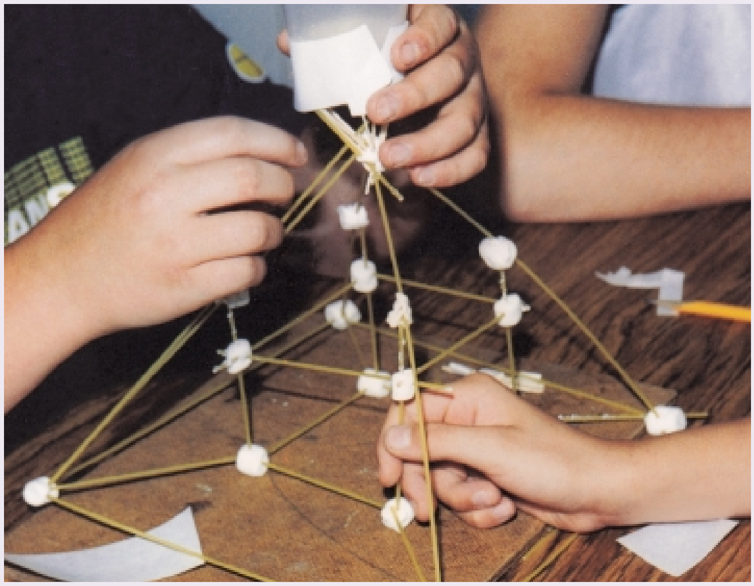


FIGURE 2

This student organized her solution to the problem into a chart and explained the process.

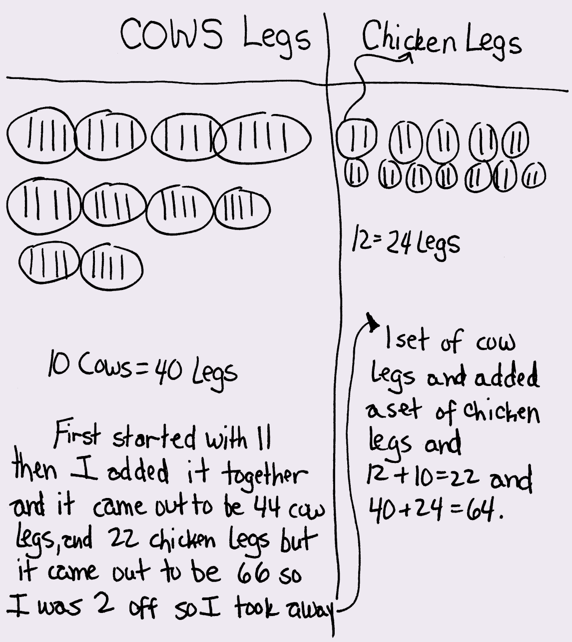
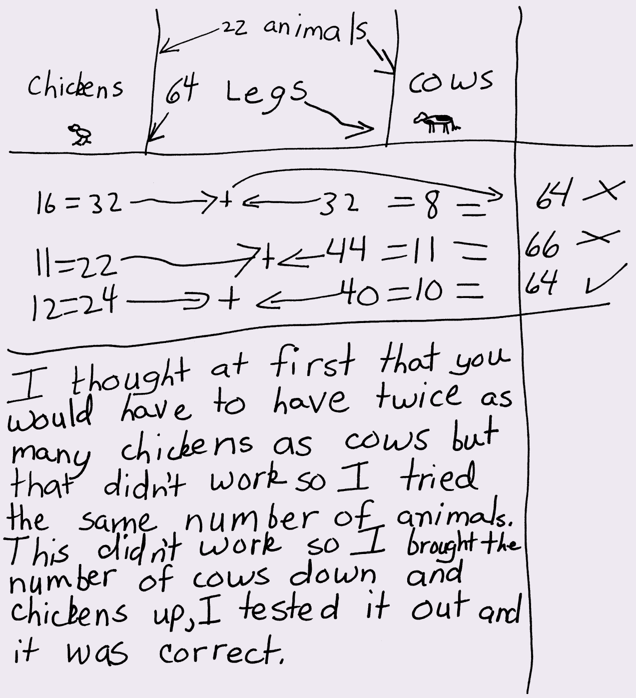


FIGURE 3

This student explained his rationale for his first attempt and his method for thinking through the solution.



the problems dumb, they were angry at me for making them feel incompetent. They did not see a mathematical puzzle as a challenge that they had the power to solve. The puzzle acted as a further challenge to their self-confidence. By resorting to self-protective disruptions, these students were threatening me, and I felt incompetent. I had lost control of my classroom. I would have to try something that they knew. Instead of assigning tasks that called for thinking and problem solving, which would require them to take risks and accept more than one possible answer, we would stick to the basic, safe computation that they asked for.

Fortunately for the students and for me, my strategy of giving up did not last long. Over the weekend, the teacher in me won out over the adult who needed to regain control. I turned to *Teaching for Thinking: Theories, Strategies, and Activities for the Classroom* (Raths et al. 1986), a book that I had disliked in college because it seemed to state the obvious. As I watched these fifth graders, however, I became more aware of a basic idea that we as educators take for granted; that is, that children will learn to think as a byproduct of all the activities, assignments, and so on that we ask them to do. The authors of *Teaching for Thinking* give us the following reminder:

> If we are to think, we must dare to think. Daring implies confidence in ourselves and in our abilities. When we have confidence, we often succeed in doing tasks far beyond our expectations. When confidence is missing, we fail at tasks that seem well within our grasp. Confidence grows largely as a result of experience. (p. 166)

What these students needed was confidence, which would come from experience. How could I reward them with beneficial experiences of broader, more difficult, and more autonomous problem solving so that they might *want* to confront a problem like the spaghetti tower?

Formulating a Plan

That weekend in September, I began planning a daily curriculum that emphasized building the children's confidence in themselves as problem solvers and creators and helping them realize the power in daring to think. We began with short tasks that we could revisit and in which the goals were to find and share mathematical solutions, working with just enough classroom structure so as not to cause meltdowns.

A magic square is an example of a task that I had used successfully with other groups, but I knew that this group would not initially be able to approach such a problem. The mathematical computation required would not in itself be a barrier. Each row, column, and diagonal must have the same sum. If the target sum is 15, for example, the students must find and arrange sets of numbers that add to 15. These students could easily generate the equations 1 + 2 + 12, 3 + 4 + 8, and 5 + 6 + 4, but they lacked experience with the mathematical processes required to begin to fill in a magic square. My students who had approached this task confidently in the past had significant experience using problem-solving strategies. They would have recognized that this problem called for such strategies as list making, trial and error, and persistence in manipulating numbers. Successful students would also recognize that the problem had no one right answer and accept that some time and patience might be required to arrive at a solution. Students in past classes had been in environments since kindergarten that modeled the use of physical objects to help solve problems. When faced with this task, some of those students would have independently sought out plastic cubes to help them model the problem and physically manipulate a solution. For this fifth-grade group to gain such confidence and self-reliance would require extensive teacher modeling and experience with problem solving.

The first time I gave this class the puzzle, I filled in seven of the nine boxes (see **fig. 4**); the students' only task was to fill in two missing numbers. Most succeeded quickly and seemed pleased with their accomplishment. This success boosted students' confidence when they tried the next modification, a square with all but four numbers filled in. Although this version required a few more calculations, it, too, offered success. This intermediate step also helped reluctant problem solvers to better understand the tasks by seeing possible solutions. The following day, we used an enlarged version of the magic square and manipulated cubes in the boxes to solve the puzzle. We counted out the target number for each row of the square, then manipulated the cubes until we had even rows, columns, and diagonals. Having succeeded at these accessible versions of the puzzle, the students found that the more open-ended task was within their reach. The objective of these small steps was not for the students to master the magic square but for them to experience repeated successes in problem solving. My hope was that by accumulating small successes, they would build confidence, perhaps even overconfidence, to approach more difficult problems.

With structure, encouragement, and time, the students found solutions to this puzzle and others. Even the most tentative mathematicians stood up at the overhead projector and "taught" their solutions. This sharing was an essential part of the group's progress; hearing others' solutions reinforced the idea that many different ways can be found to solve a problem. I also hoped that as they listened to one another's strategies, the students would get new ideas for approaches to future problems. When this group started fighting over who got to be the mathematics teacher for the day (and explain his or her strategies), we seemed to be on the right track.

FIGURE 4

A magic-square activity

Place a different number in each empty box so that all rows, columns, and diagonals have the same sum.

In October, my new district brought in a staff development professional to model effective mathematics teaching. Teachers traveled to another school in the district to observe this trainer as he gave a group of fifth graders the task of planning a day at an amusement park. I watched those students enthusiastically giving their opinions, and I agreed philosophically that this method was an ideal way to teach mathematics, yet I also knew that the same task given to my group would bring tears and anger. Having seen glimpses of progress in a short time, however, I believed that with carefully planned successes, my students could build the confidence to approach this task. We continued to work on magic squares and even some short exercises in logic (which fifth graders find much more appealing if it is called *algebra*). I learned to model and teach more strategies that I had previously taken for granted, including drawing pictures and making charts. Supported by daily opportunities to work together and share strategies, more frequent hands-on experiences, and what at times felt like too much teacher guidance, the students were ready by November to try and succeed at planning a day at the amusement park, a task that would have sent even the teacher home crying in September.

This activity was similar to previous experiences that I had given the students, in that it required them to read and interpret data, use charts or drawings to record partial solutions, and accurately compute problems using money and time. This challenge was different, though, because it required teams of students to make decisions as a group. Each group of four was given an amusement-park map that included prices and times required for each activity. Working with a specified budget and time constraints, the students were to plan an ideal day at the park. The mathematical difficulty came in accurately computing the amount of money that the group would spend and in adding blocks of time to their schedules. The class's enthusiasm during this task and their concern for accuracy indicated to me that all the shorter ventures into real-life problems and open-ended questions had given individual students sufficient success to approach this complex task with confidence. When a student who had smashed marshmallows and crumpled papers in September reported confidently that planning this hypothetical trip was "fun and pretty easy," I knew that his sense of mathematics *and of himself* were improving.

In addition to planning trips, students made up their own logic puzzles and developed more specific language when they explained solutions. By December, they were even excited by the difficult puzzle of determining whether a five-pound bag of birdseed contains one million seeds. Some explanations were far from accurate; for example, Olivia estimated that the bag would have 1,000 seeds because she knew that that amount was "a real lot." I was beginning to see evidence of understanding and articulation, however, coupled with excitement and self-reliance. Lilly explained her team's plan:

> First find out how many seeds are in one ounce. Then find out how many ounces are in a pound. Then figure out how many ounces are in five pounds. And then on a calculator add up the amount of birdseed there is in 5 lbs.

By midyear, I had gained more perspective on the students' abilities and needs. In my journal, I wrote that teaching children to be critical thinkers requires an environment of high expectations and challenges but not the absence of opportunities for developing self-confidence.

> While we've made good progress, we're far from there. . . . The first week in January, I gave them the job of designing a scale map of a town. They reacted much as they had to assignments at the beginning of the year. The measurement, the maps, the fractions, the language of conversion, and the open-endedness of the assignment were too much for them all at once. . . . Fortunately I know them well enough now to know that their shutdown is temporary. With a bit more foundation laid and schema built, we'll try it again next week.

Even with setbacks, the class was making progress. The students were beginning to accept the idea that a problem could have more than one correct answer. They were not so quick to give up when asked to try a different strategy. They were even showing signs of listening to and supporting one another's attempts. In September, some of these students had considered mathematics to be an inscrutable set of rules that the teacher dispensed; they were now able to share and critique one another's strategies. Declaring that some methods were easier to understand and that others were more efficient, the children were now seeing mathematics as a process that they could understand and participate in.

Assessment for Thinking

During this time as I reinvented myself as a teacher, I was faced with yet another stumbling block: I had to give grades and traditional report cards. For many of my students, these assessments served only to reinforce their belief that they could not succeed. Again, finding a way to get around this institutional barrier made the students and me stronger. Although I was ultimately required to give the student grades, I focused on alternative assessments while in class. Developing rubrics together as a class helped the students to understand those qualities that distinguish exemplary work. I structured self-assessments that asked the students to reflect on their effort and understanding, questions they still had, strategies they used, and their abilities to cooperate with others. I used teacher-student dialogue journals to help ensure that I acknowledged the students' successes and growth. The dialogue journals also allowed me to address goals and areas of need in a nonthreatening manner. Children often see traditional grades as being judgments imposed from on high, as yet another area of their lives over which they have no control. These student-centered assessments became tools to help students take control of their learning and to see themselves as decision makers with power. Students received the message that they were trusted and that their voices were valuable.

Conclusion

At times throughout the year, I asked myself why I forced the students to confront open-ended challenges and hands-on problem solving when these activities did not seem successful with this group. In my journal in April, I grappled with the issues that made this learning environment so important, *especially* with this group:

> Monday morning . . . . Jodie came in this morning in tears; her dad was supposed to get out of jail this weekend, but didn't. . . . Seth is coming to terms with the fact that they are going to be moving his brother away from his foster mother. . . . Matthew was caught stealing again. Does the fact that he is hungry alter how we confront him? . . . This morning when Megan finally joined the group, she told them that she was pregnant. The fact that it is not true does not make the comment any less disruptive or disturbing. Sometimes I wonder how they make it through a day.

I could easily have concluded that my students had too many personal problems to be successful at the cooperative, open-ended problem-solving tasks that I gave them. I also faced pressure from people outside the classroom who wanted to know why I took time for building and inventing when these students did not know "the basics." Both of these stances are defeatist. Once we say that some children are not capable thinkers or problem solvers, we have all but guaranteed that they will not be so. If, instead, we recognize that students gain confidence in themselves through experience with problem solving, we will be surprised by what our "low achievers" accomplish. Children desperately need the chance to explore, create, and resolve conflicts with one another if they are to move beyond the cycle of failure.

I used to take for granted that whatever questions I posed would require students to think. My fifth graders taught me that if they were to be successful, I would need to plan more carefully. Every day, teachers must structure environments, tasks, and activities that balance anxiety-ridden challenges with opportunities for success. These challenges are as important for at-risk students as they are for more advantaged populations.

What will I ask my students to do tomorrow? I do not know, but if it works, I will require them to reflect on the choices they made and the strategies they used to achieve success. If the activity does not work, if the papers get crumpled, if arguments begin or materials get abused, I may give up on the activity but not on my students.

As teachers, we should be continually amazed by what our students can do. We will be surprised by how they solve problems or tackle tasks in ways that we never imagined. We will be compelled to run and tell our colleagues when children come up with designs that we ourselves would not have thought of. If teachers do not experience this wonder, then we are not asking enough of our students.

Reference

Raths, Louis E., Selma Wasserman, Arthur Jonas, and Arnold Rothstein. *Teaching for Thinking: Theories, Strategies and Activities for the Classroom.* New York: Teachers College Press, 1986. ▲