The Roll Out Fractions Game: Comparing Fractions

The expectation that all students should use models, benchmarks, and equivalent forms to judge the size of fractions is clearly stated in *Principles and Standards for School Mathematics* within the Number and Operations Standard for grades 3–5 (NCTM 2000). The game presented in this article provides for students' development of these skills in a motivational and flexible way. Students will be able to use different strategies to compare fractions and analyze the relationships among fraction size.

General Rules

The objective of the Roll Out Fractions game is to allow students to form and compare fractions. The game is intended for two players. First, the players decide who will start the game. Then the players take turns rolling the two number cubes (see **fig. 1**). After every roll, each player decides how to use the digits obtained as numerator and denominator and tries to form the smallest fraction possible. Each player also writes his or her selected fraction in the space provided on the game board (**fig. 2**). The students then compare the fractions, writing the "greater than" symbol (>) (the phrase "is more than" was initially used to make it easier for students to understand), "less than" symbol (<), or "equal to" symbol (=), as appropriate, in the space between the two fractions. The player who creates the smallest fraction earns a point for that round (see scoring spaces on the right side of **fig. 2**). The player who won the previous round starts the next round. If the fractions are equal or equivalent, both players earn one point for that round, and the winner of the previous round begins a new round. The students follow the same procedures for each round of the game. The player with the most points at the end of five rounds wins. If a tie occurs after five rounds, both players win.

Copies of the game and scoring board were provided to the students to play the game. This served as a record of the fractions that the students used and a way to check for correctness of the students' responses.

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Important Considerations and Observations

The fourth-grade students participating in this game were familiar with whole numbers, their different representations, and whole-number operations using materials such as base-ten blocks and number lines. The students were also familiar with the following terms and concepts: *denominator, numerator, fraction bar, improper fractions, proper fractions, mixed numbers,* and *multiplication and division of whole numbers.* At least some informal knowledge of or familiarity with these concepts helped them understand different ways of compar-

ing fractions, such as multiplying or dividing the numerator and denominator by the same number to find equivalent fractions.

For this article, the emphasis of the students' experiences with the game was on using different learning levels for representing fractions included in the game: concrete, pictorial, and abstract. The concrete level involves the use of manipulative materials such as fraction tiles to represent fractional parts (**fig. 3**). The pictorial level, also known as the representational level, involves the use of representations such as charts to represent and visualize fractional parts (**fig. 4**). Finally, the abstract level involves writing or reading symbols or words to denote fractions and the relationship between fractional parts.

These learning levels could be used in isolation or in combination. For example, the following is a situation involving the concrete and abstract learning levels: Using words or symbols, the teacher asks (abstract level) a student to identify (concrete level) one-half of a whole using the fraction tiles, and the student selects (concrete level) the red tile from the fraction tiles (identified as one of two equal pieces). Similarly, at the pictorial level, the teacher asks a student to identify one-half of a whole using the fraction chart, and the student identifies (pictorial level) the yellow portion on the chart representing this amount. At the abstract level, the teacher asks the student to write the symbol for half of a whole, and the student writes "1/2" to symbolically signify this quantity.

The students were familiar with concrete, pictorial, and abstract representations of fractions and were not only allowed but encouraged to use these during the game. For example, as illustrated in **figure 5,** when comparing 1/2 and 1/4 the students could compare the fractions concretely by using the white (used as a guide), red, and blue fraction tiles. They could compare them pictorially by using the fraction chart to locate and compare the size of each fraction. They could also compare them abstractly by using the idea that 1/2 is larger than 1/4 because it is the same amount of equal pieces (the same numerator) of a larger piece (2 indicates that we are dividing a region into two equal pieces, and 4 indicates that we are dividing the same region into more partitions and, as a result, smaller equal pieces).

Also, with fraction tiles (concrete level), observe that 1/2 = 2/4 = 4/8 (see **fig. 6**). In this case, 1 white tile (a guide to represent 1 whole), 1 red tile (1/2), 2 blue tiles (2/4), and 4 yellow tiles (4/8) are used to represent the fractions. Similarly, with the fraction chart (pictorial level), the students could look for 1/2 and notice that 2/4 and 4/8 have equivalent

Figure 1

Number cube templates for Roll Out Fractions game

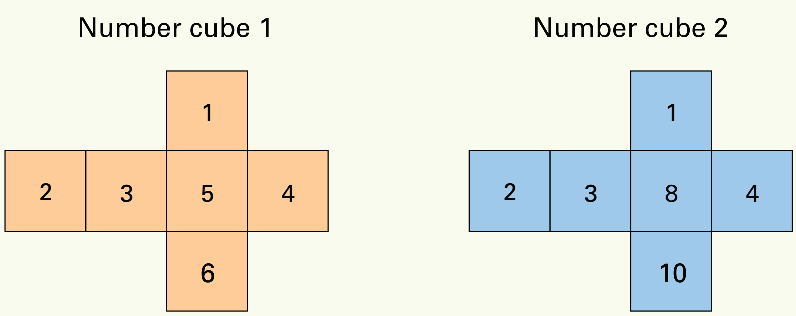


Figure 2

Roll Out Fractions game and scoring board

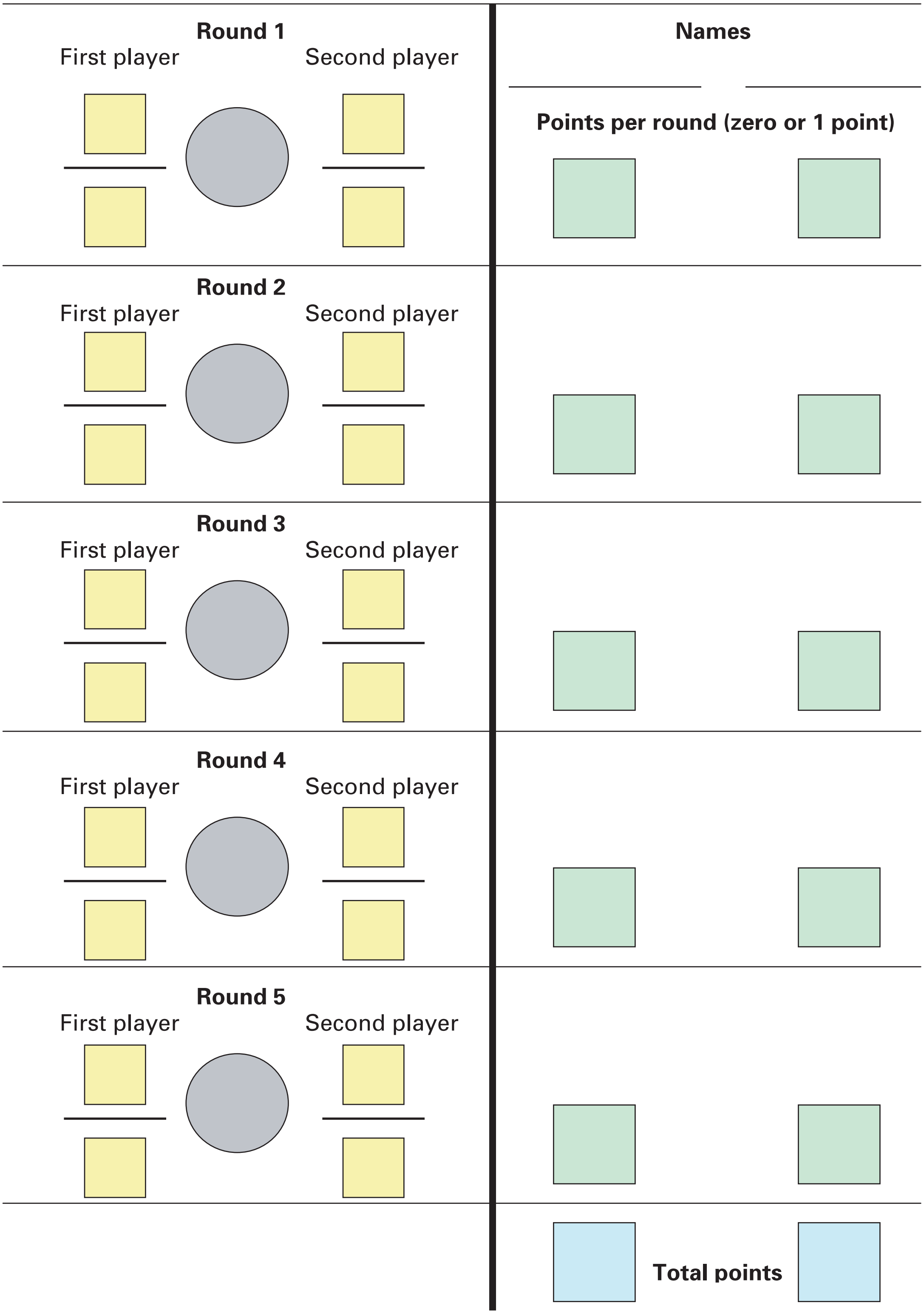
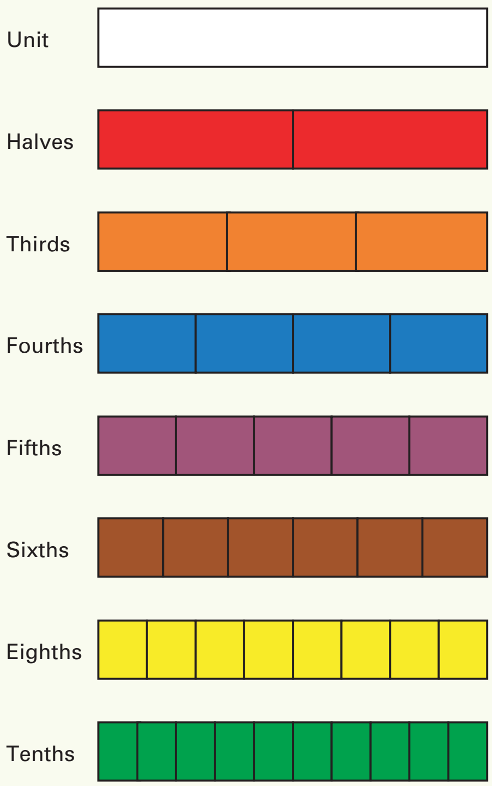


Figure 3

Fraction tiles



sizes. Pictorially, students would be able to determine rows of equivalent fractions. **Figure 7** provides examples of both the teacher's and students' moves for different learning-level combinations. Notice that labels on the fractional parts were not used on the fraction tiles or fraction chart. In order to find how many parts of the same size were equivalent to the representation presented by the teacher, students had to identify fractional values of the different partitions by using their knowledge of fractions as part of a region. Also, notice that the focus is on familiar fractions: halves, thirds, fourths, fifths, sixths, eighths, and tenths (NCTM 2000).

Students' Work During the Game

A closer look at students' strategies as they played the game revealed interesting information about their thinking processes and the effective implementation of this game. After playing several times, the students noticed that the game included some specific fraction combinations (only the more familiar fractions were selected and incorporated). Some students, when playing as the second player during a round of the game, started to make predictions of the fractions they needed to win a game. For example, if the first player's fraction was 1/8, the second player would say, "I need something like 1/9 or 1/10 to win the game." However, eventually the student would notice that fractions such as 1/9 are not possible in this game because 9 is not included as a possible outcome for the number cubes. Also, fractions such as these are not included as part of the fraction tiles or the fraction chart; in other words, the fraction tiles and chart include only the proper fractions involved in the game as a result of the different number cube combinations. Making these predictions helped the students develop a better sense of the relationships among fractions. This type of prediction could be facilitated and encouraged by asking the students to make predictions of the desired values before rolling the number cubes. For example, ask the second player during a turn, "What values do you need in order to make a smaller fraction and win the game?" and "How do you represent these fractions using fraction tiles or the fraction chart?" Notice that these questions help reinforce students' understanding of fractions at different cognitive levels. In this case, the student could verbally indicate the possible values (abstract level) and then provide the representation using the fraction tiles (concrete level) or fraction chart (pictorial level).

Another student presented a different strategy as she played the second turn of round 5 of a tied

Figure 4

Fraction chart

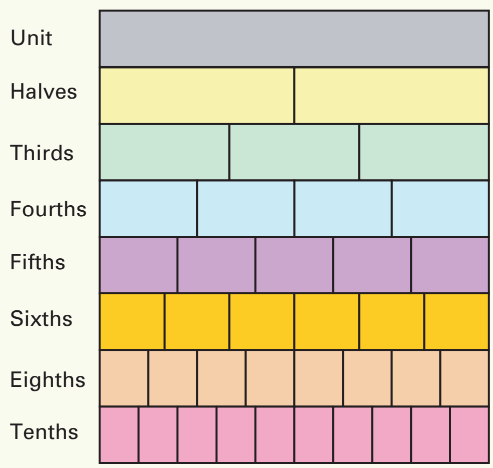


Figure 5

Representation used for comparing 1/2 and 1/4

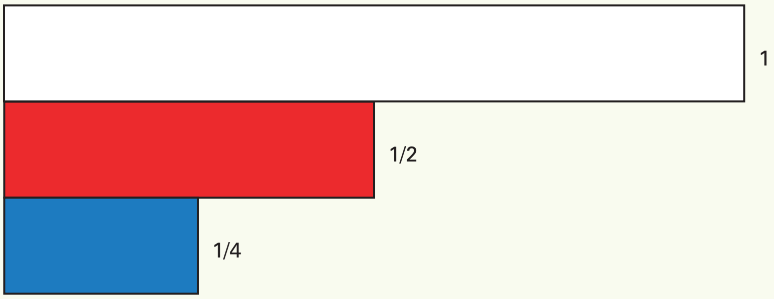
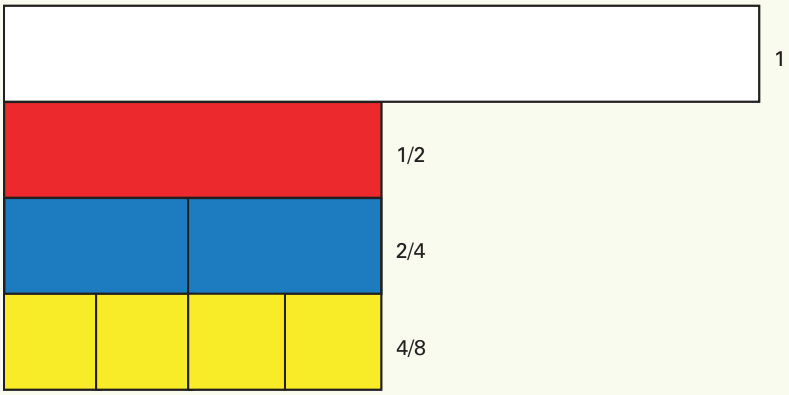


Figure 6

Representation used for comparing 1/2, 2/4, and 4/8



game. The first player's fraction was 1/3. The second player wanted to win or at least tie the game in this last round. The question was, "What values would you like to have to tie or win the game?" She first looked through the fraction chart (pictorial level) to see what values she would like to have and then wrote down fractions (abstract level) such as 1/3 or 2/6 to tie the game and 1/4, 1/5, 1/6, 1/8, 2/8, 1/10, 2/10, or 3/10 to win the game. She could have used the fraction tiles in a similar manner to find the possible fractions. Some students, with or without using fraction tiles or the chart, might notice that 1/4, 1/5, 1/6, and 1/8 are smaller than 1/3 because they have the same number of pieces (same numerator) of smaller portions (as the denominator gets larger).

After the students looked at the possible fractions, their observations included the following:

- "1/10 is the least of the fractions and best choice to win a round of a game."
- "The largest possible proper fraction is 4/5."
- "One whole was the largest fraction and worst move" (if the other improper fractions are not considered).
- "A fraction equivalent to 1/2 was the most frequent possibility" (with five possible ways to get 1/2).
- "The largest possible fraction is 10/1" (if all improper fractions are considered).
- "It is better to use a proper fraction [instead of an improper fraction] if you want to win a round of a game."
- "Only a specific number of proper fractions is possible."

The students also noticed that, for each turn, they could obtain the same number for both number cubes and form only a whole unit (for example, rolling 2 for both number cubes allows only 2/2) or roll different numbers and form either an improper fraction or a proper fraction (for example, rolling 2 for one number cube and 4 for the other allows for 2/4 or 4/2). One student, when referring to the selection of the denominator of a fraction, made the following generalization: "It is better to always use the larger number as the denominator and form fractions less than a whole." This generalization is valid because it describes a proper fraction, which is less than one whole, and provides a better chance for winning the game.

Another student strategy for comparing fractions was using benchmarks. This strategy will usually be accomplished at the abstract level without using models such as fraction tiles or a chart, but it is based on students' prior work with these types of models at the concrete and pictorial levels. The students who used this strategy compared fractions such as 2/5 and 5/8 by comparing each with 1/2 (without using the fraction tiles or the chart). In this case, the student would say, "Two-fifths are a little less than one-half, and five-eighths are a little more that one-half; then five-eighths [in some cases the student thought about 4/8 as being equivalent to 1/2 and 5/8 as being a bit more than that] should be greater than two-fifths" (NCTM 2000). This was a feasible and acceptable strategy for comparing fractions; however, during the game the students were encouraged to find and explore many different ways to compare fractions, including more abstract strategies (see the following paragraph). The idea was to help students understand the

Figure 7

Examples of the teacher's and students' moves

Students' Responses \ Teacher's Moves	Concrete	Pictorial	Abstract
	Concrete to Concrete Teacher shows the student 1 red tile (representing 1/2 **concretely**) and asks which is smaller than this tile; student selects 1 blue tile (representing 1/4 **concretely** without using words or symbols).	**Pictorial to Concrete** Teacher shows the student 1 yellow portion on fraction chart (representing 1/2 **pictorially**) and asks which is smaller than this amount; student selects 1 blue tile (representing 1/4 **concretely** without using words or symbols).	**Abstract to Concrete** Teacher asks (talks or writes, **abstractly**) the student to show a tile smaller than 1/2 (without showing fraction tiles or chart); student selects 1 blue tile (representing 1/4 **concretely** without using words or symbols).
	Concrete to Pictorial Teacher shows the student 1 red tile (representing 1/2 **concretely**) and asks which is smaller than this tile; student points to a blue portion on the fraction chart (representing 1/4 **pictorially** without using words or symbols).	**Pictorial to Pictorial** Teacher shows the student 1 yellow portion on fraction chart (representing 1/2 **pictorially**) and asks which is smaller than this amount; the student points to a portion on the fraction chart smaller than 1/2, then to a blue portion on the fraction chart (representing 1/4 **pictorially** without using words or symbols).	**Abstract to Pictorial** Teacher asks (talks or writes without showing fraction tiles or chart, **abstractly**); the student points to a portion on the fraction chart smaller than 1/2, then to a blue portion on the fraction chart (representing 1/4 **pictorially** without using words or symbols).
	Concrete to Abstract Teacher shows the student 1 red tile (representing 1/2 **concretely**) and asks which is smaller than this tile; the student writes symbols or discusses/talks to indicate 1/4 as the answer (without using fraction tiles or chart, **abstractly**).	**Pictorial to Abstract** Teacher shows the student 1 yellow portion on fraction chart (representing 1/2 **pictorially**) and asks which is smaller than this amount; the student writes symbols or discusses/talks to indicate 1/4 as the answer (without using fraction tiles or chart, **abstractly**).	**Abstract to Abstract** Teacher asks (talks or writes, **abstractly**) the student to show a tile smaller than 1/2 (without showing fraction tiles or chart); the student writes symbols or discusses/talks to indicate 1/4 as the answer (without using fraction tiles or chart, **abstractly**).

structure of fractions and the relationship among fractions in order to work with the fractions flexibly (NCTM 2000). An important note is that before using this strategy at the abstract level, the students, by using fraction tiles, had a good idea of the size of 2/5 compared with 1/2 and the size of 5/8 compared with 1/2. **Figure 8** illustrates this relationship through the use of fraction tiles; the white tile (1) is used as a guide and the red tile (1/2) as the benchmark to compare 2 purple (2/5) and 5 yellow (5/8) tiles. Similarly, at the pictorial level, the same relationship can be analyzed by using the fraction chart (see **fig. 4**): 1 yellow portion (1/2) as the benchmark, 2 purple portions (2/5), and 5 tan portions (5/8).

Some students exhibited abstract-level strategies to compare fractions without using tools such as fraction tiles or charts. For example, when comparing 3/4 with 2/3, the fractions were renamed by using the same denominator (a similar approach is presented in Tucker, Singleton, and Weaver 2002, p. 214):

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Because 9/12 > 8/12, then 3/4 > 2/3. In this case, the students knew that they could get new names for fractions by multiplying the numerator and denominator by the same numbers and that this is just another way of multiplying the fraction by a form of 1; therefore, 3/4 = 9/12 and 2/3 = 8/12. The students should notice that they are just changing

Figure 8

Representation used to compare 2/5 and 5/8 in relation to the 1/2 benchmark

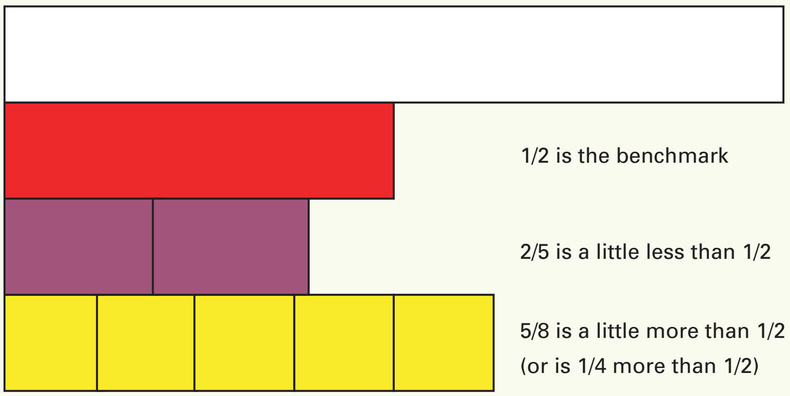


Figure 9

Students' list of proper fractions, organized from smallest to largest

1/10
1/8
1/6
1/5 = 2/10
1/4 = 2/8
3/10
1/3 = 2/6
3/8
2/5 = 4/10
1/2 = 2/4
= 3/6
= 4/8
= 5/10
3/5 = 6/10
5/8
2/3 = 4/6
3/4 = 6/8
4/5

the fractions to a common denominator in order to make the comparison.

Adaptations and Extensions

The students also adapted the rules so that the student with the larger fraction (rather than the lower fraction) earns one point. This adaptation provided a new challenge and a new perspective on fractions involving improper fractions. The possible improper fractions are the reciprocal of the proper fraction found before plus those equal to one whole, for a total of 24 combinations. These improper fractions could be converted into their corresponding mixed and whole numbers.

Another simple adaptation of the game is possible by just changing the values included for the number cubes or by adding a third one. For example, you could use 1, 2, 3, 4, 5, and 6 for one number cube and 1, 2, 3, 4, 8, and 16 for the other number cube (or add the latter as a third number cube). This adaptation will provide other combinations and allow practice with different fractions, such as 1/16 and 3/16. With an extra number cube, the students will have to make more comparisons before deciding which is the best move during a turn. For example, if the student rolls 1, 3, and 4, respectively, during a turn, then he or she must decide between 1/3, 1/4, and 3/4 as options, without including improper fractions.

What are all the possible proper fractions? This question provided a different challenge and an interesting investigation after the students played the game a number of times. The students used a combination of methods—including fraction tiles, a fraction chart, and other methods—to find all the possible combinations. Working in small groups, the class found the following answers, which the students organized in order from smallest to largest (see **figure 9**).

Altogether, they could form 26 possible common fractions (15 if we counted groups of equivalent fractions only once). The findings were presented and discussed by the whole class.

Conclusions

Students' understanding and solid grasp of fraction concepts in this game, as demonstrated by both their actions and their words, are very important factors before beginning a more rigorous or formal study of operations on fractions. The attention to the prerequisite skills and concepts and the overall experience provided by the Roll Out Fractions game will result in significant benefits in later instruction involving fraction operations and application of these skills. Students will have a better understanding of fractions at different learning levels—concrete, pictorial, and abstract—and will make decisions about how to compare them more effectively.

The students participating in the development of this article were fourth graders. However, the game experiences could be adapted to meet students' levels of understanding of fractions and other needed concepts. For example, third-grade students could concentrate on comparing the fractions at a more concrete and informal level by emphasizing the use of fraction tiles. The transition to the abstract level (writing or reading symbols or words) might take a bit longer for some students. Furthermore, in some cases, the terminology does not need to be formal or definitional. A more formal and definitional use of these terms will develop eventually as the students read, hear, use, and play with these fraction concepts and skills.

References

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Tucker, Benny F., Ann H. Singleton, and Terry L. Weaver. *Teaching Mathematics to All Children: Designing and Adapting Instruction to Meet the Needs of Diverse Learners.* Upper Saddle River, NJ: Pearson Education, 2002. ▲